Indian Statistical Institute, Bangalore
B. Math (II)
Second semester 2003-2004
Midterm Examination : Statistics (II)

Date: 04-03-2004 | Maximum Score 60 | Duration: 3 Hours

1. In the game of bowling a **strike** is made when a full setup of 10 pins is knocked down with the first delivery. Adithya's coach to monitor his progress keeps record of Adithya's performance. During one of the practice sessions in n independent trials, $X_1, X_2, ..., X_n$, the record of strike being made or not was kept. Assume that the ability to make strike is constant over trials and that in a given trial Adithya makes a strike with probability θ, $\theta \in (0,1)$. Find sufficient statistics for θ based on $X_1, X_2, ..., X_n$. Find the Fisher Information $I(\theta)$ in the sample $X_1, X_2, ..., X_n$ about the parameter θ. Give an unbiased estimator for θ. Check whether it attains *CRLB*. If yes, is it *UMVUE* for θ? If not, obtain *UMVUE* for θ and check whether it attains *CRLB*.

[08]

2. Number of incoming calls to the EPBX, handled by Smt. Rathnamma, in one unit of time may be modelled by *Poisson*(λ), $\lambda > 0$. Smt. Rathnamma is interested in estimating $\tau(\lambda) = e^{-\lambda}$ the probability of **NO** calls in one unit of time. Let $X_1, X_2, ..., X_n$ denote the number of incoming calls in one unit of time recorded on n different occasions. Can you assist Smt. Rathnamma to estimate the required probability $\tau(\lambda) = e^{-\lambda}$? Give an unbiased estimator for $\tau(\lambda)$. Is your estimator *UMVUE* for estimating $\tau(\lambda)$? If yes, substantiate. If no, obtain *UMVUE* for $\tau(\lambda)$.

[10]

3. In Que 2, if Smt. Rathnamma has reasons to believe that λ has a *prior distribution* given by *Gamma*(a, b), $a > 0$, $b > 0$ known. Obtain *posterior distribution* of λ given the observations. $X_1, X_2, ..., X_n$. Obtain *posterior* mean and variance. Suggest *Bayes estimator* for λ.

[10]

4. Suppose that a particle counter is imperfect and independently detects each incoming particle with probability θ. Suppose further that the distribution of N, the number of incoming particles in a unit of time is *truncated Poisson*(λ), given by $p(n|\lambda) = \frac{e^{-\lambda}\lambda^n}{n!(1-e^{-\lambda})}$, $n = 1, 2, ...; \lambda > 0$ is known. Let X be the number of particles detected by the counter. Show that (X, N) is minimal sufficient and N is ancillary for θ. Is the estimator $W = \frac{X}{N}$ unbiased for θ? Obtain its *MSE*.

[10]

5. Let $X_1, X_2, ..., X_n$ be a random sample from the *pdf* $f(x|\theta) = e^{-(x-\theta)}$; $-\infty < \theta < x < \infty$. Show that $X_{(1)}$ is a complete sufficient statistic for θ. Use *Basu's Theorem* to show that $X_{(1)}$ and S^2 are independent.

[10]

6. *Pareto distribution* is often appropriate to model income. Let $X_1, X_2, ..., X_n$ be a random sample from *Pareto density* given by $f(x|\theta) = c\theta^c x^{-(c+1)}$ for $x \geq \theta$ and 0 otherwise, where $\theta > 0$ and c is a positive constant. Find *maximum likelihood estimator* for θ based on $X_1, X_2, ..., X_n$. Obtain *method of moments* estimator for θ and check whether it is consistent.

[08]

7. Let $X_n, Y_n, n \geq 1$ be sequences of random variables and X be a random variable, all defined on the same probability space, such that $X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{p} c$, where c is a finite constant. Prove that $X_nY_n \xrightarrow{d} cX$.

[10]